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SURFACE WAVE PROPAGATION IN A LIQUID-SATURATED POROUS LAYER OVERLYING A HOMOGENEOUS TRANSVERSELY ISOTROPIC HALF-SPACE AND LYING UNDER A UNIFORM LAYER OF LIQUID

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Abstract—Dispersion of Rayleigh-type surface waves is studied in a liquid-saturated porous solid layer under a uniform layer of homogeneous liquid, and lying over a transversely isotropic elastic half-space. Special cases have been deduced by reducing the depth of the layer to zero and by changing the transversely isotropic solid to an isotropic elastic solid.

A frequency equation in the form of a tenth-order determinant is obtained. For numerical calculations, a particular model consisting of a water-saturated sandstone layer lying over a beryl† solid and under a uniform layer of water is considered. To observe the effects of the depths of the layers on the phase velocity, dispersion curves for the phase velocity have been plotted for different values of the ratio of the depths of two layers.

INTRODUCTION

Liquid-saturated porous rocks are often present on and below the surface of the Earth. Sedimentary layers consisting of sandstone or limestone saturated with water, are usually present below oceans. Layers of porous solids such as sandstone or limestone saturated with ground water or oil are present in the Earth's crust. Constitutive equations and equations of motion, including inertial terms, for such solids were formulated by Biot (1956a, b). Biot (1956a, b, 1962a, b) found that propagation of two dilatational waves along with one shear wave is possible in such solids. In the absence of dissipation, these waves are elastic in nature, the propagation being at constant velocity with undiminished amplitude. If dissipation is taken into account, each of the waves is dispersive and dissipative; that is, the velocity is a function of frequency, and amplitude undergoes spatial attenuation. Deresiewicz (1960, 1961, 1964a, b, 1965), Deresiewicz and Rice (1962) and Deresiewicz and Levy (1967) investigated various aspects of the effects of the presence of boundaries on the propagation of plane harmonic seismic waves in liquid-saturated porous solids. Deresiewicz (1960, 1961, 1964a, b, 1965), Deresiewicz and Rice (1962) and Deresiewicz saturated porous solids.

There are reasonable grounds for the assumption that geologic materials are anisotropic. An obvious example is that of the materials deposited in water. Anisotropy in the Earth's crust and upper mantle have significant effects on the surface wave characteristics such as phase and group velocities. Many investigators have studied the propagation of elastic waves in an isotropic medium. Stoneley (1926), Biot (1952) and Tolstoy (1954) studied the propagation of elastic waves in a system consisting of a liquid layer of finite depth overlying an isotropic half-space. Abubaker and Hudson (1961) studied the dispersive properties of liquid overlying a semi-infinite, homogeneous, transversely isotropic half-space. Gogna (1979) considered surface wave propagation in a homogeneous anisotropic layer lying over a homogeneous, isotropic, elastic half-space and under a uniform layer of liquid.

Here we have considered the problem (two-dimensional) of surface wave propagation in a liquid-saturated porous solid layer, overlying an impervious, transversely isotropic, elastic, solid half-space and under a uniform layer of liquid. This appears to be of practical

† Beryl is a hexagonal crystal of the class specified by the group D_{6h}^{1} (Love, 1944).

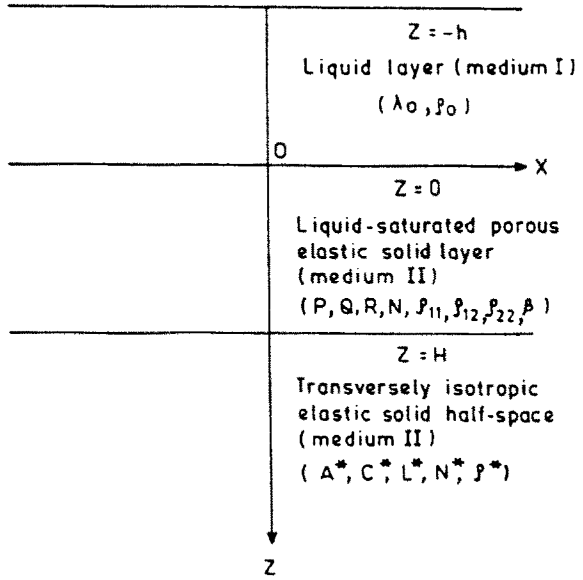

Fig. 1.

interest as the sediments deposited under water may be assumed to be transversely isotropic. It is also a more realistic model for the ocean bottom. Hence, it is relevant to the study of Rayleigh waves at the upper surface of the ocean. Some special cases have also been discussed.

FORMATION OF THE PROBLEM

We consider a medium consisting of a liquid-saturated porous layer, of thickness H, resting on a transversely isotropic elastic half-space and under a uniform layer of liquid, of thickness h. We consider a rectangular coordinate system, such that the z-axis is chosen in the direction of increasing depth and $z = 0$ is taken as the interface between the two layers. Hence, the transversely isotropic elastic solid (medium III) occupies the region $z > H$, the liquid-saturated porous solid (medium II) occupies the region $0 < z < H$, and the region $-h < z < 0$ is occupied by the liquid layer (medium I), as shown in Fig. 1.

We discuss a two-dimensional problem with wave front parallel to the y-axis, so that the displacement components in the x and z direction are independent of y, and the components in the y direction will vanish.

BASIC EQUATIONS AND THEIR SOLUTIONS

For the liquid layer (medium I), the equation of motion in terms of the displacement potential ϕ_0 is given by

$$\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial z^2} = \frac{1}{\alpha^2}\frac{\partial^2 \phi_0}{\partial t^2}, \tag{1a}$$

where $\alpha(=\sqrt{\lambda_0/\rho_0})$ is the velocity of the dilatational wave in the liquid, ρ_0 is the density and λ_0 is the bulk modulus of the liquid.

The displacement components u_0, w_0 and pressure p are given by

$$u_0 = \frac{\partial \phi_0}{\partial x}, \quad w_0 = \frac{\partial \phi_0}{\partial z} \quad \text{and} \quad p = -\sigma_{zz} = -\lambda_0 \nabla^2 \phi_0, \tag{1b}$$

where σ_{zz} is the normal component of stress in the liquid.

Assuming $\phi_0 = \bar{\phi}_0(z)\,e^{ik(x-ct)}$, substituting in (1a) and solving, yields

$$\bar{\phi}_0(z) = A_0\,e^{kz\xi_0} + B_0\,e^{-kz\xi_0}$$

and therefore

$$\phi_0 = (A_0\,e^{kz\xi_0} + B_0\,e^{-kz\xi_0})\,e^{ik(x-ct)}; \tag{1c}$$

where A_0, B_0 are arbitrary constants and $\xi_0 = \sqrt{1-c^2/\alpha^2}$.

For the liquid-saturated porous solid (medium II), the field equations are given by Biot (1962a) as

$$\left.\begin{aligned} N\nabla^2\mathbf{u} + \mathrm{grad}\,\{(D+N)e + Q\varepsilon\} &= \frac{\partial^2}{\partial t^2}\{\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}\} + b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}) \\ \text{and} \qquad \mathrm{grad}\,\{Qe + R\varepsilon\} &= \frac{\partial^2}{\partial t^2}\{\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}\} - b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}), \end{aligned}\right\} \tag{2}$$

where $e = \mathrm{div}\,\mathbf{u}$ and $\varepsilon = \mathrm{div}\,\mathbf{U}$.

u and **U** are displacements in the solid and liquid parts of the porous aggregate, respectively; D, N, Q and R are the elastic constants for the solid–liquid aggregate; and ρ_{11}, ρ_{12} and ρ_{22} are the dynamical coefficients.

The dissipation coefficient b is

$$b = \frac{\eta}{\chi}\beta^2 \tag{3}$$

where η is the fluid viscosity, χ is the coefficient of permeability and β is the porosity.

This expression for b is valid for the low frequency range, where the flow in the pores is of Poiseuille-type. For higher frequencies, a correction factor is applied to the viscosity, replacing it by ηF, where F is a complex function of frequency evaluated by Biot (1956b).

We consider the Helmholtz resolution of each of the two displacement vectors, in the form

$$\left.\begin{aligned} \mathbf{u} &= \mathrm{grad}\,\phi + \mathrm{curl}\,\mathbf{H}, \\ \mathbf{U} &= \mathrm{grad}\,\psi + \mathrm{curl}\,\mathbf{G}. \end{aligned}\right\} \tag{4}$$

Substituting (4) into eqns (2) yields a pair of equations which are satisfied provided that

$$\left.\begin{aligned} P\nabla^2\phi + Q\nabla^2\psi &= \frac{\partial^2}{\partial t^2}\{\rho_{11}\phi + \rho_{12}\psi\} + b\frac{\partial}{\partial t}(\phi-\psi); \\ Q\nabla^2\phi + R\nabla^2\psi &= \frac{\partial^2}{\partial t^2}\{\rho_{12}\phi + \rho_{22}\psi\} - b\frac{\partial}{\partial t}(\phi-\psi); \end{aligned}\right\} \tag{5}$$

and

$$\left.\begin{aligned} N\nabla^2\mathbf{H} &= \frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{H} + \rho_{12}\mathbf{G}) + b\frac{\partial}{\partial t}(\mathbf{H}-\mathbf{G}); \\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{H} + \rho_{22}\mathbf{G}) - b\frac{\partial}{\partial t}(\mathbf{H}-\mathbf{G}). \end{aligned}\right\} \tag{6}$$

If we eliminate ψ from eqns (5), we shall obtain a fourth-order differential equation in ϕ. To solve this equation, we substitute

$$\phi = \phi_1 + \phi_2 \tag{7}$$

and obtain

$$\left\{\nabla^2 + \left(\frac{\omega}{\alpha_j}\right)^2\right\}\phi_j = 0, \quad (j = 1, 2) \tag{8}$$

where ϕ_1 and ϕ_2 are time harmonic scalar potentials;

$$\alpha_1^2 = \frac{B+\sqrt{B^2-4AC}}{2C}, \quad \alpha_2^2 = \frac{B-\sqrt{B^2-4AC}}{2C}$$

$$A = (D+2N)R-Q^2,$$

$$B = \left(\rho_{11}+\mathrm{i}\frac{b}{\omega}\right)R+\left(\rho_{22}+\mathrm{i}\frac{b}{\omega}\right)(D+2N)-2\left(\rho_{12}-\mathrm{i}\frac{b}{\omega}\right)Q,$$

$$C = \left(\rho_{11}+\mathrm{i}\frac{b}{\omega}\right)\left(\rho_{22}+\mathrm{i}\frac{b}{\omega}\right)-\left(\rho_{12}-\mathrm{i}\frac{b}{\omega}\right)^2; \tag{9}$$

and ω is the angular frequency.

With the help of eqns (7) and (8), it can be found that

$$\psi = \mu_1\phi_1 + \mu_2\phi_2, \tag{10}$$

where

$$\mu_j = \frac{(\rho_{11}\omega+\mathrm{i}b)R-(\rho_{12}\omega-\mathrm{i}b)Q-(A/\alpha_j^2)}{(\rho_{22}\omega+\mathrm{i}b)Q-(\rho_{12}\omega-\mathrm{i}b)R} \quad (j = 1, 2). \tag{11}$$

Solving eqns (6), we obtain

$$\mathbf{G} = -\left(\frac{\rho_{12}\omega-\mathrm{i}b}{\rho_{22}\omega+\mathrm{i}b}\right)\mathbf{H}$$

$$= \alpha_0\mathbf{H}(\text{say}) \tag{12}$$

and

$$\left\{\nabla^2 + \left(\frac{\omega}{\alpha_3}\right)^2\right\}\mathbf{H} = 0, \tag{13}$$

where

$$\alpha_3^2 = \frac{N\left(\rho_{22}+\mathrm{i}\frac{b}{\omega}\right)}{C}. \tag{14}$$

Hence, in an unbounded liquid-saturated, porous medium, two dilatational waves can propagate along with one shear wave.

For two-dimensional motion in the x–z plane, the displacements in the solid $\mathbf{u} = (u, 0, w)$ and in the liquid $\mathbf{U} = (U, 0, W)$ are given by

$$\left.\begin{aligned} u &= \frac{\partial\phi_1}{\partial x}+\frac{\partial\phi_2}{\partial x}+\frac{\partial\psi_1}{\partial z}, \\ w &= \frac{\partial\phi_1}{\partial z}+\frac{\partial\phi_2}{\partial z}-\frac{\partial\psi_1}{\partial x}, \\ U &= \mu_1\frac{\partial\phi_1}{\partial x}+\mu_2\frac{\partial\phi_2}{\partial x}+\alpha_0\frac{\partial\psi_1}{\partial z}, \\ W &= \mu_1\frac{\partial\phi_1}{\partial z}+\mu_2\frac{\partial\phi_2}{\partial z}-\alpha_0\frac{\partial\psi_1}{\partial x}; \end{aligned}\right\} \tag{15}$$

where $\psi_1 = (-\mathbf{H})_y$.

Stresses in the solid σ_{ij}, and in the liquid σ, are

$$\left.\begin{aligned} \sigma_{ij} &= (De+Q\varepsilon)\delta_{ij}+2N\varepsilon_{ij}, \\ \sigma &= Qe+R\varepsilon, \end{aligned}\right\} \tag{16}$$

where δ_{ij} is the Kronecker delta, and

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right). \tag{17}$$

Wave potentials ϕ_1, ϕ_2 and ψ_1 are the solutions of eqns (8) and (13), respectively, and may be written as

$$\left.\begin{aligned} \phi_j &= [A_j e^{kz\xi_j}+B_j e^{-kz\xi_j}]\,e^{ik(x-ct)}, \qquad (j=1,2) \\ \psi_1 &= \{A_3 e^{kz\xi_3}+B_3 e^{-kz\xi_3}\}\,e^{ik(x-ct)}; \end{aligned}\right\} \tag{18}$$

where A_j, B_j are arbitrary constants, k is the wave number, c is the phase velocity and

$$\xi_j = \sqrt{1-(c^2/\alpha_j^2)} \quad (j = 1,2,3). \tag{19}$$

For a homogeneous, transversely-isotropic, elastic solid (medium III) with symmetry about the z-axis, following Love (1944), the strain energy volume density function has the form

$$2W_0^* = A^*(e_{xx}^2+e_{yy}^2)+C^*e_{zz}^2+2F^*(e_{xx}+e_{yy})e_{zz} \\ +2(A^*-2N^*)e_{xx}e_{yy}+L^*(e_{yz}^2+e_{xz}^2)+N^*e_{xy}^2; \tag{20}$$

where the displacement $\mathbf{u}^* = (u^*, v^*, w^*)$, and

$$e_{i,j} = \begin{cases} \dfrac{\partial u_i^*}{\partial x_j}+\dfrac{\partial u_j^*}{\partial x_i}, & (i \neq j) \\[2ex] \dfrac{\partial u_i^*}{\partial x_i}, & (i = j). \end{cases} \tag{21}$$

Restricting motion to two dimensions (x, z), the strain energy volume density function (20) becomes

$$2W_0^* = A^*e_{xx}^2+c^*e_{zz}^2+2F^*e_{xx}e_{zz}+L^*e_{zx}^2. \tag{22}$$

Since W_0^* is of positive definite form, then

$$A^* > 0, \quad C^* > 0, \quad L^* > 0 \quad \text{and} \quad A^*C^* > F^{*2}. \tag{23}$$

It is also assumed that $A^* > L^*$ and $C^* > L^*$.

Components of stress can be derived by the formulae:

$$\sigma_{ij}^* = \frac{\partial W_0^*}{\partial e_{ij}} \quad (i, j = x, y, z). \tag{24}$$

The equations of motion where there are no body forces, are

$$\left.\begin{aligned} A^*\frac{\partial^2 u^*}{\partial x^2} + L^*\frac{\partial^2 u^*}{\partial z^2} + (F^* + L^*)\frac{\partial^2 w^*}{\partial x\,\partial z} &= \rho^*\frac{\partial^2 u^*}{\partial t^2}, \\ L^*\frac{\partial^2 w^*}{\partial x^2} + C^*\frac{\partial^2 w^*}{\partial z^2} + (F^* + L^*)\frac{\partial^2 u^*}{\partial x\,\partial z} &= \rho^*\frac{\partial^2 w^*}{\partial t^2}; \end{aligned}\right\} \tag{25}$$

where ρ^* is the density of the transversely isotropic medium.

As in medium II, we seek a solution of (25) of the form

$$(u^*, w^*) = [U^*(z), W^*(z)]\, e^{ik(x-ct)}, \tag{26}$$

and find that eqns (25) reduce to

$$\left.\begin{aligned} L^*U^{*\prime\prime} + ik(F^* + L^*)W^{*\prime} - (A^* - \rho^*c^2)k^2U^* &= 0, \\ C^*W^{*\prime\prime} + ik(F^* + L^*)U^{*\prime} - (L^* - \rho^*c^2)k^2W^* &= 0; \end{aligned}\right\} \tag{27}$$

where the primes denote differentiation with respect to "z".

Following the orthodox method of solving simultaneous linear equations with constant coefficients, we write

$$\left.\begin{aligned} U^* &= P^* e^{-ksz}, \\ W^* &= Q^* e^{-ksz}. \end{aligned}\right\} \tag{28}$$

Substituting these values into (27), we obtain

$$\left.\begin{aligned} (s^2L^* + R^*)P^* - (isJ^*)Q^* &= 0, \\ -(isJ^*)P^* + (s^2C^* + S^*)Q^* &= 0; \end{aligned}\right\} \tag{29}$$

where $J^* = F^* + L^*$, $R^* = \rho^*c^2 - A^*$ and $S^* = \rho^*c^2 - L^*$.

For the non-trivial solution of eqns (29), we have

$$L^*C^*s^4 + (R^*C^* + S^*L^* + J^{*2})s^2 + R^*S^* = 0. \tag{30}$$

Equation (30) is quadratic in s^2 and has the roots

$$\frac{-\Gamma \mp (\Gamma^2 - 4L^*C^*R^*S^*)^{1/2}}{2L^*C^*}, \tag{31}$$

where $\Gamma = R^*C^* + S^*L^* + J^{*2}$.

The ratio of the displacement components U_j^* and W_j^*, from (29), corresponding to $s = s_j$, is

$$\frac{W_j^*}{U_j^*} = \frac{Q_j^*}{P_j^*} = \frac{L^* s_j^2 + R^*}{\mathrm{i} s_j J^*} = m_j \text{ (say)}. \tag{32}$$

Thus, the solutions of eqns (25) can be written as

$$\begin{aligned} u^* &= \{P_1^* e^{-ks_1 z} + P_2^* e^{-ks_2 z} + P_3^* e^{ks_1 z} + P_4^* e^{ks_2 z}\} e^{ik(x-ct)}, \\ w^* &= \{m_1(P_1^* e^{-ks_1 z} + P_3^* e^{ks_1 z}) + m_2(P_2^* e^{-ks_2 z} + P_4^* e^{ks_2 z})\} e^{ik(x-ct)}; \end{aligned} \tag{33}$$

where P_1^*, P_2^*, P_3^* and P_4^* are arbitrary constants and

$$s_j^2 = \frac{-\Gamma + (-1)^j (\Gamma^2 - 4L^* C^* R^* S^*)^{1/2}}{2L^* C^*} \quad (j = 1, 2). \tag{34}$$

Since the displacement components tend to zero when z tends to infinity, we therefore take the expressions for u^* and w^* as

$$\begin{aligned} u^* &= (P_1^* e^{-ks_1 z} + P_2^* e^{-ks_2 z}) e^{ik(x-ct)}, \\ w^* &= (m_1 P_1^* e^{-ks_1 z} + m_2 P_2^* e^{-ks_2 z}) e^{ik(x-ct)}; \end{aligned} \tag{34a}$$

where s_1 and s_2 are assumed to be real and positive.

BOUNDARY CONDITIONS

For two-dimensional motion, we consider the boundary conditions appropriate for the following:

(a) The free surface of the liquid layer, which is the vanishing of the normal stress component at $z = -h$, i.e.

(i) $$-(p)_{\mathrm{I}} = (\sigma_{zz})_{\mathrm{I}} = \lambda_0 \nabla^2 \phi = 0. \tag{35}$$

(b) The interface between the liquid layer and the liquid-saturated porous solid. Following Deresiewicz and Skalak (1963), these are the continuity of the stress components, liquid pressure and component of velocity normal to the interface averaged over the bulk area, along the interface at $z = 0$, i.e.

$$\left.\begin{aligned} &\text{(i)} && (\sigma_{zz})_{\mathrm{II}} + (\sigma)_{\mathrm{II}} = -(p)_{\mathrm{I}} = \lambda_0 \nabla^2 \phi \\ &\text{(ii)} && (\sigma_{zx})_{\mathrm{II}} = 0 \\ &\text{(iii)} && \frac{1}{\beta}(\sigma)_{\mathrm{II}} = -(p)_{\mathrm{I}} = \lambda_0 \nabla^2 \phi \\ &\text{(iv)} && (1-\beta)(\dot{w})_{\mathrm{II}} + \beta(\dot{W})_{\mathrm{II}} = \dot{w}_0. \end{aligned}\right\} \tag{36}$$

(c) The interface between the liquid-saturated porous solid and the transversely isotropic

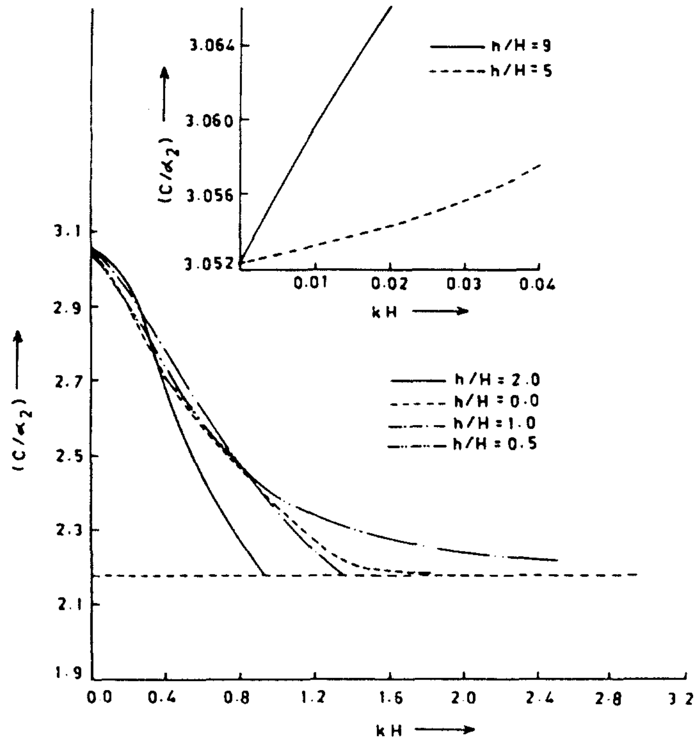

Fig. 2.

elastic solid. Following Deresiewicz and Skalak (1963), assuming the transversely isotropic elastic solid to be impervious, these are the continuity of stress and displacement components, and the vanishing of the normal velocity of the liquid relative to the solid in the liquid-saturated porous solid along the interface at $z = -H$, i.e.

$$\left.\begin{array}{ll}\text{(i)} & (\sigma_{zz})_{II}+(\sigma)_{II} = (\sigma^*_{zz})_{III}\\ \text{(ii)} & (\sigma_{zx})_{II} = (\sigma^*_{zx})_{III}\\ \text{(iii)} & (w)_{II} = (w^*)_{III}\\ \text{(iv)} & (u)_{II} = (u^*)_{III}\\ \text{(v)} & (\dot{w})_{II}-(\dot{W})_{II} = 0.\end{array}\right\} \quad (37)$$

Making use of (1), (15), (16), (18), (24) and (34a) in the above boundary conditions, we obtain 10 homogeneous equations in A_0, B_0, A_1, B_1, A_2, B_2, A_3, B_3, P^*_1 and P^*_2. The non-trivial solution of this system of equations requires

$$|a_{ij}| = 0 \quad (38)$$

where a_{ij}, the entries of the tenth-order square matrix, are as follows:

$$a_{11} = a_{12} = 2-\frac{T_1}{N}\frac{c^2}{\alpha_1^2}, \qquad a_{13} = a_{14} = 2-\frac{T_2}{N}\frac{c^2}{\alpha_2^2},$$

$$a_{15} = a_{16} = 2\xi_3, \qquad a_{17} = a_{18} = 0,$$

$$a_{19} = a_{110} = \frac{\lambda_0}{N}\frac{c^2}{\alpha^2}, \qquad a_{21} = -a_{22} = 2\xi_1,$$

$$a_{23} = -a_{24} = 2\xi_2, \qquad a_{25} = -a_{26} = 1+\xi_3^2,$$

$$a_{2j} = 0 \; (j = 7, 8, 9, 10), \qquad a_{31} = a_{32} = \frac{Q + R\mu_1}{N}\frac{c^2}{\alpha_1^2},$$

$$a_{33} = a_{34} = \frac{Q + R\mu_2}{N}\frac{c^2}{\alpha_2^2}, \qquad a_{3j} = 0 \; (j = 5, 6, 7, 8),$$

$$a_{39} = a_{310} = -\beta\frac{\lambda_0}{N}\frac{c^2}{\alpha^2}, \qquad a_{41} = -a_{42} = (1 - \beta + \beta\mu_1)\xi_1,$$

$$a_{43} = -a_{44} = (1 - \beta + \beta\mu_2)\xi_2, \qquad a_{45} = -a_{46} = 1 - \beta + \beta\alpha_0,$$

$$a_{47} = a_{48} = 0, \qquad a_{49} = -a_{410} = \xi_0,$$

$$a_{i1} = a_{n1}e^{kH\xi_1}, \qquad a_{i2} = a_{n2}e^{-kH\xi_1},$$

$$a_{i3} = a_{n3}e^{-kH\xi_2}, \qquad a_{i4} = a_{n4}e^{-kH\xi_2},$$

$$a_{i5} = a_{n5}e^{-kH\xi_3}, \qquad a_{i6} = a_{n6}e^{-kH\xi_3},$$

$$(n = i - 4; \quad i = 5, 6)$$

$$a_{57} = P_1 e^{-s_1 kH}, \qquad a_{58} = P_2 e^{-s_2 kH},$$

$$a_{59} = a_{5,10} = 0, \qquad a_{67} = R_1 e^{-s_1 kH},$$

$$a_{68} = R_2 e^{-s_2 kH}, \qquad a_{69} = a_{6,10} = 0,$$

$$a_{71} = \xi_1 e^{kH\xi_1}, \qquad a_{72} = -\xi_1 e^{-kH\xi_1},$$

$$a_{73} = \xi_2 e^{kH\xi_2}, \qquad a_{74} = -\xi_2 e^{-kH\xi_2},$$

$$a_{75} = e^{kH\xi_3}, \qquad a_{76} = -e^{kH\xi_3},$$

$$a_{77} = -Q_1 e^{-s_1 kH}, \qquad a_{78} = -Q_2 e^{-s_2 kH},$$

$$a_{79} = 0 = a_{7,10}, \qquad a_{81} = e^{kH\xi_1},$$

$$a_{82} = -e^{-kH\xi_1}, \qquad a_{83} = e^{kH\xi_2},$$

$$a_{84} = -e^{-kH\xi_2}, \qquad a_{85} = \xi_3 e^{kH\xi_3},$$

$$a_{86} = \xi_3 e^{-kH\xi_3}, \qquad a_{87} = -e^{-s_1 kH},$$

$$a_{88} = -e^{-s_2 kH}, \qquad a_{89} = a_{810} = 0,$$

$$a_{91} = (1 - \mu_1)\xi_1 e^{kH\xi_1}, \qquad a_{92} = (\mu_1 - 1)\xi_1 e^{-kH\xi_1},$$

$$a_{93} = (1 - \mu_2)\xi_2 e^{kH\xi_2}, \qquad a_{94} = (\mu_2 - 1)\xi_2 e^{-kH\xi_2},$$

$$a_{95} = (1 - \alpha_0)e^{kH\xi_3}, \qquad a_{96} = (\alpha_0 - 1)e^{-kH\xi_3},$$

$$a_{9j} = 0 \; (j = 7, 8, 9, 10), \qquad a_{10j} = 0 \; (j = 1, 2, \ldots, 8),$$

$$a_{109} = e^{-kH\xi_0}, \qquad a_{1010} = e^{kH\xi_0}, \tag{39}$$

where

$$\left.\begin{aligned} T_j &= P + Q + \mu_j(Q + R), \\ P_j &= \left(F^* + C^* \cdot \frac{L^* s_j^2 + R^*}{J^*}\right)\frac{1}{N}, \\ R_j &= \frac{F^* s_j^2 - R^*}{s_j J^*}, \quad \text{and} \quad Q_j = \frac{L^* s_j^2 + R^*}{s_j J^*} \quad (j = 1, 2). \end{aligned}\right\} \tag{40}$$

The equation $|a_{ij}| = 0$, given by (38), is the required frequency equation relating the phase velocity c, to the wavelength $2\pi/k$. The wavelength is a multi-valued function of phase velocity (each value corresponding to the different mode of propagation) and hence indicates the dispersive nature of the wave. Such a surface wave will be homogeneous if, and only if, the equation given by (38) has a real solution, i.e. there should be at least one value of c for which s_1 and s_2 are real and positive.

SPECIAL CASES

(i) Substituting

$$A^* = C^* = \lambda+2\mu, \quad F^* = \lambda \quad \text{and} \quad L^* = \mu, \tag{41}$$

a transversely isotropic half-space can be changed to an isotropic half-space. Hence, we have

$$\begin{aligned} s_1^2 &= 1-\frac{c^2}{\alpha^2}, & s_2^2 &= 1-\frac{c^2}{\beta^2},\\ P_1 &= -\left(2-\frac{c^2}{\beta^2}\right), & P_2 &= -2,\\ Q_1 &= s_1, & Q_2 &= -\frac{1}{s_2},\\ R_1 &= 2s_1, & R_2 &= \left(2-\frac{c^2}{\beta^2}\right)\Big/ s_2; \end{aligned}$$

where

$$\alpha^2 = \frac{\lambda+2\mu}{\rho^*} \quad \text{and} \quad \beta^2 = \frac{\mu}{\rho^*}. \tag{42}$$

Using the above relations, the frequency equation (38) reduces to the equation obtained by Hazra (1984), as the dispersion equation for Rayleigh-type surface wave propagation in a liquid-saturated porous layer, lying over an isotropic elastic half-space and under a uniform layer of liquid. Furthermore for $kh \to \infty$, the liquid layer will behave as a liquid half-space and the reduced frequency equation will give surface wave propagation in a liquid-saturated porous layer, bounded between an isotropic elastic solid and a liquid half-space. It is found to be the same as that obtained by Hazra (1984).

(ii) Reducing the thickness of the liquid-saturated porous layer to zero, i.e. $H = 0$ (without loss of generality, we can put the porosity $\beta = 0$ also), we get

$$\begin{vmatrix} 0 & 0 & e^{-kh\xi_0} & e^{kh\xi_0} \\ P_1 & P_2 & -\dfrac{\lambda_0}{N}\dfrac{c^2}{\alpha^2} & -\dfrac{\lambda_0}{N}\dfrac{c^2}{\alpha^2} \\ R_1 & R_2 & 0 & 0 \\ Q_1 & Q_2 & \xi_0 & -\xi_0 \end{vmatrix} = 0 \tag{43}$$

as the dispersion equation of a liquid layer, overlying a transversely isotropic, elastic solid half-space, which is the same as that obtained by Abubaker and Hudson (1961). The values of P_1, P_2, Q_1, Q_2, R_1 and R_2 are as in eqn (40).

Further substituting $h = 0$, eqn (43) reduces to

$$P_1R_2 - R_1P_2 = 0, \tag{44}$$

which gives Rayleigh-type surface wave propagation at the free surface of a transversely isotropic, elastic half-space.

Substituting (42) in (44), we get

$$(2 - c^2/\beta^2)^2 = 4(\sqrt{1 - c^2/\alpha^2})(\sqrt{1 - c^2/\beta^2}), \tag{45}$$

the equation for Rayleigh wave propagation at the free surface of an isotropic elastic solid.

(iii) Removing the overlying liquid layer by putting $h = 0$, the frequency equation (38) will reduce to

$$|b_{ij}| = 0, \tag{46}$$

where b_{ij}, the entries of a square matrix of the order eight, are given by

$$b_{ij} = \begin{cases} a_{ij} & i = 1, 2, 3 \\ a_{i-1,j} & i = 5, 6, \ldots, 9 \end{cases} \quad (j = 1, 2, \ldots, 8).$$

Equation (46) is the frequency equation for surface wave propagation in a liquid-saturated porous layer lying over a transversely isotropic, elastic solid half-space. Using relations (42), the transversely isotropic, elastic half-space can be further changed to an isotropic elastic solid.

DISCUSSION AND NUMERICAL RESULTS

Since a large number of parameters enter into the final expressions, then in order to discuss the possibility of propagation of surface waves discussed above along the x direction, a particular model is considered. The model considered is assumed to consist of a layer of water-saturated sandstone under a uniform layer of water and overlying a beryl solid as the transversely isotropic, elastic half-space.

Equation (38) is a complex equation. For real wave numbers it is not possible to find the value of the real wave velocity. This equation has, therefore, been reduced to a real equation by assuming that the water-saturated sandstone is non-dissipative. We may mention that this is an assumption in order to solve the frequency equation (38) numerically, to obtain the velocity of propagation.

For this model, we calculated the ratio of the phase velocity to the velocity of slow dilatational waves in a water-saturated sandstone layer (c/α_2), for given values of the dimensionless number kH. The value of the ratio c/α_2 is found to be different for different values of kH.

For the water layer, following Ewing *et al.* (1957), for sound speed, density and bulk modulus, we have taken the following values:

$$\lambda_0 = 0.214 \times 10^{11} \text{ dynes cm}^{-2},$$

$$\rho_0 = 1 \text{ g cm}^{-3},$$

giving the velocity of the P wave as

$$\alpha = 1.463 \times 10^5 \text{ cm s}^{-1}.$$

For water-saturated sandstone (medium II), keeping in view the experimental results given by Yew and Jogi (1976) which differ slightly from the experimental results given by Fatt (1959), the following values of the relevant parameters are taken:

$$P = 2.15 \times 10^{11} \text{ dynes cm}^{-2}$$
$$Q = 0.013 \times 10^{11} \text{ dynes cm}^{-2}$$
$$R = 0.0637 \times 10^{11} \text{ dynes cm}^{-2}$$
$$N = 0.922 \times 10^{11} \text{ dynes cm}^{-2}$$
$$\rho_{11} = 1.9032 \text{ g cm}^{-3}$$
$$\rho_{12} = 0.0 \text{ g cm}^{-3}$$
$$\rho_{22} = 0.268 \text{ g cm}^{-3}$$
$$\beta = 0.268$$
$$\eta = 0 \text{ (non-dissipative solid).}$$

The velocities of the P_f, P_s and SV waves for the above constants are

$$\alpha_1 = 3.326 \times 10^5 \text{ cm s}^{-1}$$
$$\alpha_2 = 1.54 \times 10^5 \text{ cm s}^{-1}$$
$$\alpha_3 = 2.2 \times 10^5 \text{ cm s}^{-1},$$

respectively.

For the impervious, elastic beryl half-space (medium III), following Love (1944), the values of the relevant parameters are taken to be

$$A^* = 26.94 \times 10^{11} \text{ dynes cm}^{-2}$$
$$C^* = 23.63 \times 10^{11} \text{ dynes cm}^{-2}$$
$$F^* = 6.61 \times 10^{11} \text{ dynes cm}^{-2}$$
$$L^* = 6.53 \times 10^{11} \text{ dynes cm}^{-2}$$
$$\rho^* = 2.7 \text{ g cm}^{-3}.$$

h denotes the depth of the water layer and H is that of the water-saturated sandstone layer. For the ease of numerical calculation, we have fixed the value of h/H. Numerical results have been obtained only for the following values of h/H:

$$\frac{h}{H} = 0.0,\ 0.5,\ 1.0,\ 2.0,\ 5.0 \quad \text{and} \quad 9.0.$$

Using all the above values of parameters for the assumed model and for each value of h/H, we obtained solutions of eqn (38) for c/α_2 in the appropriate range (so that s_1 and s_2 remain real). The value of the dimensionless number kH is considered to vary from 0 to 3. For the solutions, a computer program in FORTRAN-IV was used on a PC.

When the depth of the water layer is zero, i.e. $h/H = 0$, the phase velocity decreases rapidly with increasing values of kH. It keeps on decreasing almost at the same rate until kH assumes the value 1.2 approximately, after which the rate of decrease of c becomes gradual. For larger values of kH, the phase velocity becomes almost constant, approaching the velocity of the P_f wave in a liquid-saturated porous solid.

For the case when $h/H = 0.5$, i.e. the thickness of the water layer is half that of the porous solid layer, the behaviour of the dispersion curve is the same as in the previous case. However, the rate of decrease in phase velocity becomes slower as kH assumes the value 1.0, approximately.

When the thickness of both layers are the same, i.e. $h/H = 1$, it was observed that the phase velocity decreases rapidly with increasing values of kH. The rate of decrease remains almost constant. The phase velocity attains a minimum value, approximately equal to the velocity of the P_f wave in water-saturated sandstone at $kH \approx 1.3$, after which the second mode starts.

If the thickness of the water layer is double that of the porous layer, the phase velocity decreases more rapidly than when $h = H$, attaining a minimum value for $kH \approx 0.9$.

It has been observed that as the value of h/H increases, the phase velocity of the surface wave decreases. However, the rate of decrease of phase velocity with increasing kH, decreases.

If the thickness of the water layer is considered larger than that of the porous solid layer, e.g. $h/H \geqslant 5$, then reverse behaviour of the dispersion curve is observed. The value of phase velocity increases with increasing values of kH and also with increasing values of h/H. A wave exists only for smaller values of kH.

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